The Voltage Boost Enabled by Luminescence Extraction in Solar Cells

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***Abstract*—Over the past few years, the application of the physical principle, i.e., "luminescence extraction," has produced record voltages and efficiencies in photovoltaic cells. Luminescence extraction is the use of optical design, such as a back mirror or textured surfaces, to help internal photons escape out of the front surface of a solar cell. The principle of luminescence extraction is exemplified by the mantra "a good solar cell should also be a good LED." Basic thermodynamics says that the voltage boost should be related to concentration ratio *C* of a resource by $\Delta V = (kT/q)\ln\{C\}$. In light trapping (i.e., when the solar cell is textured and has a perfect back mirror), the concentration ratio of photons $C = \{4n^2\}$; therefore, one would expect a voltage boost of $\Delta V = (kT/q)\ln\{4n^2\}$ over a solar cell with no texture and zero back reflectivity, where *n* is the refractive index. Nevertheless, there has been ambiguity over the voltage benefit to be expected from perfect luminescence extraction. Do we gain an open-circuit voltage boost of $\Delta V = (kT/q)\ln\{n^2\}$, $\Delta V = (kT/q)\ln\{2n^2\}$, or $\Delta V = (kT/q)\ln\{4n^2\}$? What is responsible for this voltage ambiguity $\Delta V = (kT/q)\ln\{4\} \approx 36$ mV? We show that different results come about, depending on whether the photovoltaic cell is optically thin or thick to its internal luminescence. In realistic intermediate cases of optical thickness, the voltage boost falls in between: $\ln\{n^2\} < (q\Delta V/kT) < \ln\{4n^2\}$.**

***Index Terms*—Luminescence, photovoltaic cells, solar energy.**

I. Introduction

IMPROVEMENTS to the optical design of the solar cell have recently enabled efficiencies close to the Shockley–Quiesser limit [1]. For example, by increasing the back mirror reflectivity of the solar cell, Alta Devices achieved a series of record efficiencies, culminating in the present record of 28.8% with a 1-sun single-junction gallium arsenide (GaAs) solar cell [2]. These records were mainly due to increases in the open-circuit voltages [1]. For example, in 2010, Alta Devices created a record-breaking cell of 27.6% efficiency [3] that had an open-circuit voltage 77 mV greater than the previous record, yet had 0.2 mA/cm^2 less short-circuit current [3], [4].

In a solar cell at open-circuit voltage, absorbed photons generate electron–hole pairs, which are then either radiatively reemitted or lost to nonradiative recombination. Radiative reemission can be reabsorbed by the cell or escape out of a solar cell surface. The small escape cone for a semiconductor/air interface, as described by Snell's law, makes it difficult for the photon to escape out of the front surface; it is much more likely for the luminescent photon to be lost to an absorbing back substrate. Photon emission is required as a reciprocal process to absorption. If photons are only absorbed through the front surface, emission out of the front is required, but emission out of the back is not [1], [5], [6]. Thus, we want to minimize the emission out of the back surface by adding a back mirror [1], [7], [8]. In the ideal case, we have a perfect back reflector and only radiative recombination; therefore, at open-circuit voltage, all absorbed photons are eventually reemitted out of the front surface of the cell [1], [9], [10]. In the presence of nonradiative recombination, a textured surface, along with the back mirror, can aid photons in escaping out the front of the cell. A textured surface randomizes the direction of the photon, allowing it multiple chances to get into the escape cone before parasitic absorption.

At the open-circuit voltage condition, the probability that an electron–hole pair recombines to emit a photon that escapes out the front of the cell is called the external luminescence yield, $\eta_{\rm ext}$. In the ideal case of $\eta_{\rm ext} = 1$, at open-circuit voltage, every electron–hole pair recombines radiatively, and with the aid of a perfect back reflector, all internal photons eventually escape out the front of the cell. The open-circuit voltage $V_{\rm oc}$ can be expressed as [1], [11]

$$V_{\rm oc} = V_{\rm oc,\ ideal} - \frac{kT}{q}\ln\left(\frac{1}{\eta_{\rm ext}}\right). \quad (1)$$

From (1), we see that the open-circuit voltage is penalized by $\Delta V_{\rm penalty} = -\frac{kT}{q}\ln(\frac{1}{\eta_{\rm ext}})$ in a nonideal cell with $\eta_{\rm ext} < 1$.

In [12], it was established that when a solar cell has a textured surface and a perfect back mirror, the absorption enhancement would be $4n^2$ compared with a planar solar cell with zero back reflectivity, where *n* is the refractive index of the absorbing material. Basic thermodynamics says that the voltage boost should be related to concentration ratio *C* of a resource by $\Delta V = (kT/q)\ln\{C\}$; therefore, we should obtain a voltage boost of $\Delta V = (kT/q)\ln\{4n^2\}$ when we texture and add a back mirror to the solar cell. Equivalently, $\Delta V_{\rm penalty} = -(kT/q)\ln\{4n^2\}$ for a planar cell with zero back reflectivity. It was also shown in [13] that $\eta_{\rm ext} = \frac{1}{4n^2}$ for a planar solar cell with no back mirror, yielding the same $\Delta V_{\rm penalty}$. More recently, however, it was shown in [14] that for a thick strongly absorbing solar cell with only radiative recombination, $\eta_{\rm ext} = \frac{1}{n^2+1}$, yielding $\Delta V_{\rm penalty} \approx -\frac{kT}{q}\ln\{n^2\}$. These two different expressions for $\eta_{\rm ext}$ lead to a

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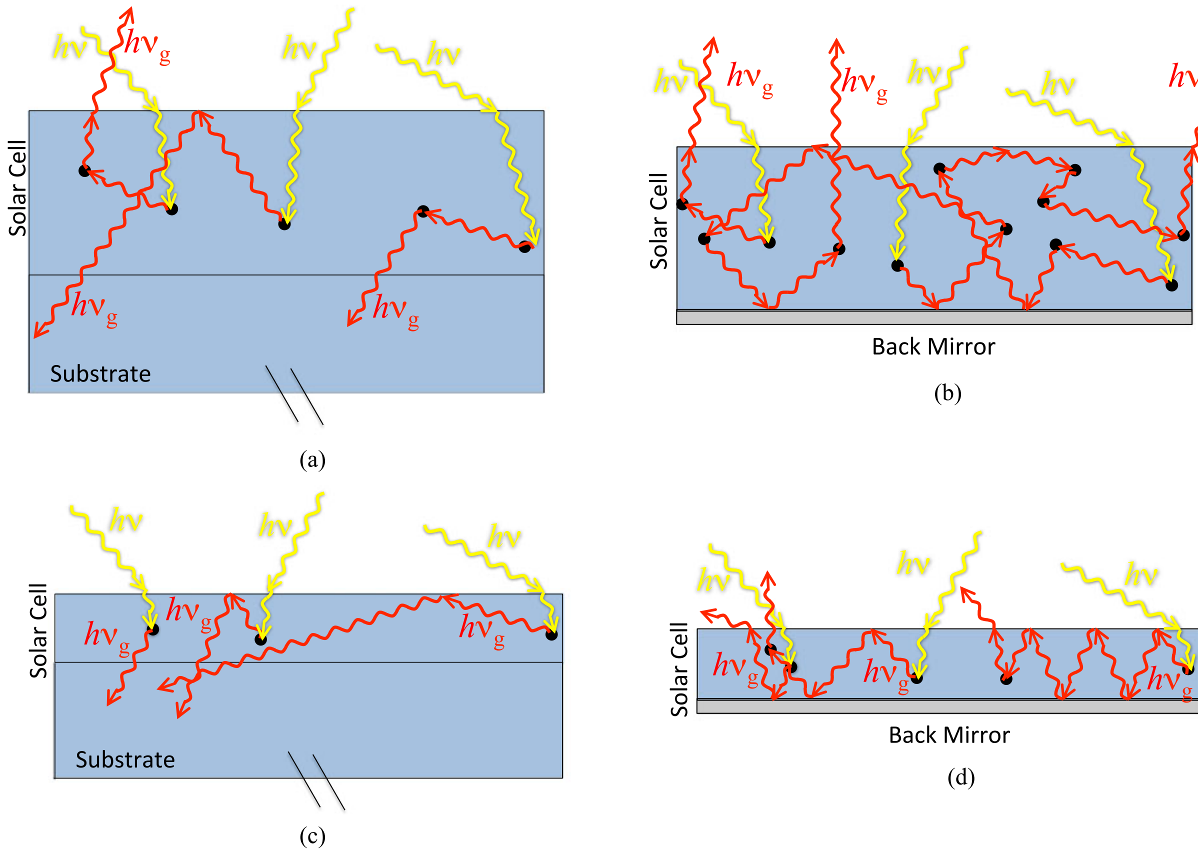

Fig. 1. Solar cells at open-circuit voltage. (a) Cell on an index-matched substrate. (b) Cell on a perfect back reflector. The incident photons are absorbed and reemitted as internal luminescence. In (a), it is difficult for the internal photons to escape out of the front surface due to total internal reflection; most of the photons are lost to the bottom substrate. In (b), due to the perfect back reflector, all the photons are eventually emitted out of the front surface. In (a) and (b), the cells are optically thick to the internal luminescence. In (c) and (d), the cells are optically thin to the internal photons.

voltage discrepancy of $(kT/q)\ln\{4\}\approx 36$ mV. What is the correct $\Delta V_{\text{penalty}}$? Here, we show that different results come about depending on whether the photovoltaic cell is optically thin or thick to its internal luminescence. We derive the correct $\Delta V_{\text{penalty}}$ in both limiting cases of optical transparency and intermediate cases with real material properties, resolving all factors of 2.

II. Limiting Cases of Luminescence Extraction

Luminescence extraction is the escape of internal photons out of the front surface of a solar cell. Both texturing the surface of the solar cell and adding a back mirror can improve luminescence extraction. Fig. 1(a) shows a planar untextured solar cell on an index-matched back substrate. Externally incident photons, shown in yellow, refract toward the normal upon entrance into the cell and are absorbed. If the cell is at open-circuit voltage and is an ideal material with only radiative recombination, the photon is reemitted (shown in red), at a lower energy, around the bandgap energy. We assume that the angular distribution of reemitted photons is isotropic. This internal photon eventually escapes out of the front surface or the back surface (more likely, as the back substrate is index-matched). The reemitted photon is of lower energy than the absorbed photon, and we can distinguish two types of cell: 1) a cell that is "optically thick" to the internal photon, meaning that the internal photon may have many chances for reabsorption before escape; or 2) a cell that is "optically thin" to the internal photon, meaning that the internal photon is rarely reabsorbed before escape. It is important to note that even if a cell is optically thin to the internal photon, it can be strongly absorbing of the higher energy externally incident photons. Fig. 1(a) and (b) shows the case of a material that is optically thick to the internal photon, with Fig. 1(b) schematically depicting the photon dynamics when a perfectly reflecting back mirror is included. Fig. 1(c) and (d) shows the case of a cell that is optically thin to the internal luminescence, without and with a back mirror, respectively.

In [14], the external luminescence yield η_{ext} is expressed as

$$\eta_{\text{ext}} = \frac{\eta_{\text{int}} P_{\text{esc}}}{1 - \eta_{\text{int}} P_{\text{abs}}} \tag{2}$$

where η_{int} is the internal luminescence yield, P_{esc} is the average probability that an internally emitted photon escapes the front of the cell without reabsorption, and P_{abs} is the average probability that an internally emitted photon is reabsorbed. The internal luminescence yield η_{int} is defined as the ratio of radiative recombination to total recombination per unit volume [1].

We now derive η_{ext} for planar solar cells with no back mirror, in the limits of weak and strong absorption of the internal luminescence. We assume only radiative recombination, a perfect antireflection coating on the cell, and an index-matched substrate below. The key physical difference between the two cases

TABLE I
EXTERNAL LUMINESCENCE YIELD $\eta_{\rm ext}$ FOR LIMITING CASES OF THE OPTICAL DESIGN

	(1) Optically thick, only radiative recombination	(2) Optically thin, only radiative recombination	(3) Optically thin, nonradiative recombination dominates
(a) Texture + perfect back mirror	$\eta_{\rm ext} = 1$	$\eta_{\rm ext} = 1$	$\eta_{\rm ext} = \eta_{\rm int}$
(b) Texture + air back mirror	$\eta_{\rm ext} = 1/2$	$\eta_{\rm ext} = 1/2$	$\eta_{\rm ext} = \eta_{\rm int}/2$
(c) Planar + air back mirror	$\eta_{\rm ext} = 1/2$	$\eta_{\rm ext} = 1/2$	$\eta_{\rm ext} = \eta_{\rm int}/(4n^2)$
(d) Planar + perfect back mirror	$\eta_{\rm ext} = 1$	$\eta_{\rm ext} = 1$	$\eta_{\rm ext} = \eta_{\rm int}/(2n^2)$
(e) Planar + absorbing substrate	$\eta_{\rm ext} = 1/(n^2+1)$	$\eta_{\rm ext} = 1/(4n^2)$	$\eta_{\rm ext} = \eta_{\rm int}/(4n^2)$

is that in the thin cell, we assume that the angular distribution of internal photons striking the front surface from below is approximately uniform, whereas in the thick cell, the distribution of the internal photons is strongly affected by the absorption in the bulk and is assumed to be approximately Lambertian [15].

We first look at the limiting case where the cell is optically thin (i.e., weakly absorbing) to the internally luminescent photon energies. For this limit, we can recognize that the probability of front surface escape, relative to substrate absorption, is the fraction of solid angle that is subtended by the escape cone [12]. We can derive $\eta_{\rm ext}$ as follows [15]:

$$\eta_{\rm ext} = \frac{2\pi \int_0^{\sin^{-1}\left(\frac{1}{n}\right)} \sin\theta d\theta}{2\pi \int_0^{\pi} \sin\theta d\theta} = \frac{1}{2}\left(1 - \sqrt{1 - \frac{1}{n^2}}\right) \approx \frac{1}{4n^2}. \quad (3)$$

This limit can also be derived by noting from [14] that $P_{\rm esc} \rightarrow \frac{1}{4n^2}$ for a thin cell and $P_{\rm abs} \rightarrow 0$; therefore, from (2), $\eta_{\rm ext} \rightarrow \frac{1}{4n^2}$.

We now look at the limiting case of a material that is very strongly absorbing of the internal luminescence. To calculate this limit, note that $\eta_{\rm ext}$ can be equivalently defined as the ratio of radiative emission out the front of the cell to total loss rate of photons out of the cell [1]. We have

$$\eta_{\rm ext} = \frac{L_{\rm ext\uparrow}}{L_{\rm ext\uparrow} + L_{\rm ext\downarrow}} \quad (4)$$

where $L_{\rm ext\uparrow}$ is the radiative emission rate out of the front of the cell, and $L_{\rm ext\downarrow}$ is the emission rate out of the back of the cell and into the substrate.

At the top surface, since we assume a perfect antireflection coating, we can assume perfect transmittance of internally luminescent photons in the escape cone θ_s (given by Snell's law, $n \sin\theta_s = 1$). There is total internal reflection for internal luminescent photons outside the escape cone. Due to the many absorption events inside the material, the internal photons hitting the top surface have a Lambertian distribution. The angle-averaged transmittance of the internally luminescent photons through the top surface $T_{\rm int\uparrow}$ is, thus, given by [15]

$$T_{\rm int\uparrow} = \frac{2\pi \int_0^{\sin^{-1}\left(\frac{1}{n}\right)} \sin\theta\cos\theta d\theta}{2\pi \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta} = \frac{1}{n^2} \quad (5)$$

Where the $\cos\theta$ term accounts for the Lambertian distribution.

Since the cell is free of nonradiative recombination, the only other photon flux out of the cell is through the rear surface, which is described by the bottom luminescent transmittance $T_{\rm int\downarrow}$, and $T_{\rm int\downarrow}$ is unity because the top cell is index-matched to the substrate below. Applying (3) and (4) yields

$$\eta_{\rm ext} = \frac{T_{\rm int\uparrow}}{T_{\rm int\uparrow} + T_{\rm int\downarrow}} = \frac{1}{1+n^2} \quad (6)$$

as was found in [14] for a thick strongly absorbing cell. We have a factor of ≈ 4 difference in $\eta_{\rm ext}$ between the limits of being optically thin and optically thick to the internal luminescence. Physically, we can understand the increase in $\eta_{\rm ext}$ in the optically thick case by realizing that due to many absorption and emission events, there are more chances for the internal photon to get into the escape cone and escape out the front.

When we also consider solar cells where nonradiative recombination dominates and surface texturing is used, we can draw up Table I to summarize $\eta_{\rm ext}$ for all the limiting cases of luminescence extraction. In the calculations for this table, we assume that the distribution of electron-hole pairs and internal photons is homogeneous.

A solar cell with a perfect back reflector and only radiative recombination ($\eta_{\rm int} = 1$) has $\eta_{\rm ext} = 1$ for cells both optically thick or thin to the internal luminescence, regardless of whether surface texturing is used or not. In such a cell, the internal luminescence will continue to be reabsorbed and radiatively reemitted until it is able to escape out of the front surface of the cell.

If the perfect back reflector is replaced by an air interface on the back surface for all these cases with only radiative recombination, $\eta_{\rm ext} = \frac{1}{2}$, as the internal luminescence now sees the same escape cone at the front and back surfaces (for simplicity, we assume both top and bottom surfaces have the same antireflection coating). The result of $\eta_{\rm ext} = \frac{1}{2}$ can also be derived for each of these cases: For a planar optically thick cell, we can apply (6), with $T_{\rm int\uparrow} = T_{\rm int\downarrow} = \frac{1}{n^2}$, yielding $\eta_{\rm ext} = \frac{1}{2}$. For a planar optically thin cell, we can apply (2) and the analysis in [12]. We have $\eta_{\rm int} = 1$, $P_{\rm esc} = \frac{1}{4n^2}$ and $P_{\rm abs} = 1 - \frac{1}{2n^2}$, yielding $\eta_{\rm ext} = \frac{1}{2}$. When a random texture is added to both the optically thin and thick cells with an air back interface, in the case of only radiative recombination, we still have $\eta_{\rm ext} = \frac{1}{2}$. The random texture does not change the escape cone on average, and the randomizing effect of the texture is unnecessary, as multiple internal absorption and emission events randomize the directions of the internal photons. With or without the texture, the photons will have equal probability of escape out of the top or bottom surface of the cell. A texture only improves $\eta_{\rm ext}$ when

TABLE II
VOLTAGE PENALTY $\Delta V_{\mathrm{penalty}}$ FOR LIMITING CASES OF THE OPTICAL DESIGN

	(1) Optically thick, only radiative recombination	(2) Optically thin, only radiative recombination	(3) Optically thin, nonradiative recombination dominates
(a) Texture + perfect back mirror	$\Delta V_{\mathrm{penalty}} = 0$	$\Delta V_{\mathrm{penalty}} = 0$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(1/\eta_{\mathrm{int}})$
(b) Texture + air back mirror	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(2)$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(2)$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(2) - \frac{kT}{q} \times \ln(1/\eta_{\mathrm{int}})$
(c) Planar + air back mirror	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(2)$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(2)$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(4n^2) - \frac{kT}{q} \times \ln(1/\eta_{\mathrm{int}})$
(d) Planar + perfect back mirror	$\Delta V_{\mathrm{penalty}} = 0$	$\Delta V_{\mathrm{penalty}} = 0$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(2n^2) - \frac{kT}{q} \times \ln(1/\eta_{\mathrm{int}})$
(e) Planar + absorbing substrate	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(n^2 + 1)$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(4n^2)$	$\Delta V_{\mathrm{penalty}} = -\frac{kT}{q} \times \ln(4n^2) - \frac{kT}{q} \times \ln(1/\eta_{\mathrm{int}})$

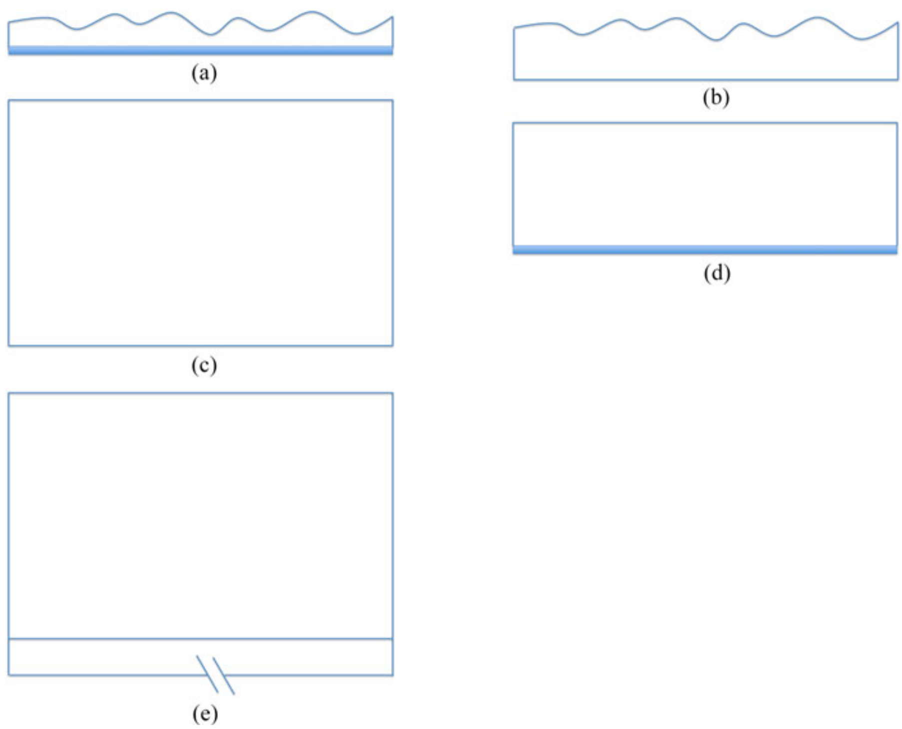

Fig. 2. Diagrams for the cases summarized in Tables I and II. (a) Textured cell with a perfect back mirror. (b) Textured cell with air on the back surface. (c) Planar cell with air on the back surface. (d) Planar cell with a perfect back mirror. (e) Planar cell with an absorbing index-matched substrate. The thicknesses of the cells in the figure are approximately scaled such that each cell has the same absorption of external incident photons and, consequently, the same short-circuit current and $V_{\mathrm{oc,\ ideal}}$.

nonradiative recombination is present, and internal absorption and emission events are subject to parasitic losses.

The last column in the table looks at a cell that is optically thin to the internal luminescence, in the case where nonradiative recombination dominates over radiative recombination. In this column, we assume $\eta_{\mathrm{int}} \ll 1$ and $P_{\mathrm{abs}} \ll 1$; therefore, we can approximate $1-\eta_{\mathrm{int}} P_{\mathrm{abs}} \approx 1$. We first look at the case of a cell with surface texturing and a perfect back mirror. If an externally incident photon is absorbed by this cell, the photon is radiatively reemitted with probability η_{int}. Since the cell is very thin, the probability of reabsorption is low, and the internal photon's direction is continually randomized by the surface texture until it can escape out the front surface. Equivalently, we have $P_{\mathrm{esc}} \approx 1$; therefore, $\eta_{\mathrm{ext}} \approx \eta_{\mathrm{int}}$ by (2). In the case of a textured cell with an air interface at the back, η_{ext} is reduced by half, as there is equal probability of escaping out the front and back surface. Using (2), with $P_{\mathrm{esc}} \approx \frac{1}{2}$, we also get $\eta_{\mathrm{ext}} \approx \frac{\eta_{\mathrm{int}}}{2}$. If we have a planar cell, again with a back air interface, the photon will only escape if it is emitted radiatively (with probability η_{int}) *and* emitted into the escape cone (probability of $\frac{1}{4n^2}$). This gives $\eta_{\mathrm{ext}} \approx \frac{\eta_{\mathrm{int}}}{4n^2}$. Equivalently, we have $P_{\mathrm{esc}} \approx \frac{1}{4n^2}$; therefore, from (2), we confirm $\eta_{\mathrm{ext}} = \frac{\eta_{\mathrm{int}}}{4n^2}$. Similarly, an absorbing index-matched back substrate has $\eta_{\mathrm{ext}} = \frac{\eta_{\mathrm{int}}}{4n^2}$. Finally, if we have a planar solar cell with a perfect back mirror, the escape cone is effectively doubled (as emission downwards into the escape cone can reflect out of the cell); therefore, $\eta_{\mathrm{ext}} = \frac{\eta_{\mathrm{int}}}{2n^2}$. This is seen from (2), with $P_{\mathrm{esc}} = \frac{1}{2n^2}$.

In Table II, we translate η_{ext} to $\Delta V_{\mathrm{penalty}}$ as given in (1). Fig. 2 diagrammatically shows the different cases summarized in Tables I and II.

In Table II, we give $\Delta V_{\text{penalty}}$ for various limiting cases of optical design. However, $\Delta V_{\text{penalty}}$ is given in reference to $V_{\text{oc, ideal}}$, and $V_{\text{oc, ideal}}$ depends on the absorption of the solar cell [1], [16]

$$V_{\text{oc, ideal}} = \frac{kT}{q} \ln\left(\frac{\int_0^\infty A(E)\, S(E)\, dE}{\int_0^\infty A(E)\, b(E)\, dE}\right) \tag{7}$$

where $A(E)$ is the absorptivity as a function of photon energy, $S(E)$ is the solar spectrum, $b(E)$ is the blackbody spectrum, and the acceptance angle of the solar cell is assumed to be the full hemisphere. As the efficiency of a solar cell depends on its absolute voltage, we can only compare the $\Delta V_{\text{penalty}}$ of different solar cell designs when $V_{\text{oc, ideal}}$ is held constant. Additionally, voltage is only part of the picture in solar cells; when comparing voltages, it only makes sense to compare voltages of cells with identical short-circuit current. Thus, we want to compare $\Delta V_{\text{penalty}}$ for cells with the same $A(E)$. For example, in Table II, we obtain $\Delta V_{\text{penalty}}$ for a cell on an absorbing substrate. This cell should be compared with a planar cell of half the thickness on a perfectly reflecting back mirror. Both cells have exactly the same absorption and short-circuit current, but the cell with a back mirror has a higher voltage. We can compare a planar cell with an absorbing substrate to a planar cell with air at the back, with the same thickness, as both have the same absorption. The removal of the absorbing substrate causes a boost in voltage, though the current is unchanged. Similarly, when comparing a planar cell on substrate against a textured cell on a perfect back mirror, the thicknesses L_{planar} and L_{textured} should approximately follow

$$A(E) = 1 - \exp\left(-\alpha(E)\, L_{\text{planar}}\right) \approx \frac{\alpha(E)\, L_{\text{textured}}}{\alpha(E)\, L_{\text{textured}} + \frac{1}{4n^2}} \tag{8}$$

where the right side is the approximate absorption of a randomly textured cell [12].

These comparisons underscore the importance of luminescence extraction. It is not enough to simply optimize the short-circuit current of a cell; attention has to be paid to the optical design to optimize the voltage as well. The $\Delta V_{\text{penalty}}$ quantifies how good the optical design of the cell is.

In [12], it was found that the concentration ratio due to light trapping (i.e., when the solar cell is textured and has a perfect back mirror) is $C = \{4n^2\}$. Due to this concentration ratio, one would expect a voltage boost $\Delta V = (kT/q)\ \ln\{4n^2\}$ over a solar cell with no texture and zero back reflectivity. We can reconcile this assertion by noting that this concentration ratio was derived for the *external* incident photons under the assumptions of a weakly absorbing material. We expect a voltage boost when the concentration ratio of the *internal* photons increases. In the case of a material with only radiative recombination, such as in the columns (1) and (2) of Table II, there is no voltage difference between a cell with a perfect back mirror and planar surface and a cell with a perfect back mirror and a textured surface. Similarly, there is no voltage difference between a planar cell with air at the back and a textured cell with air at the back. This is because even though the concentration of external photons has increased in the textured cell, the texture helps the internal photons escape. In the planar case, the internal photon must be reabsorbed several times in order to access the escape cone, leading to the same internal photon concentration as the textured case. However, in column (3) of Table II, nonradiative recombination dominates, and the probability an external photon is absorbed and reemitted radiatively is η_{int}, while the probability that an internal photon is absorbed and reemitted radiatively can be approximated as zero. Thus, in column (3), the concentration of internal photons can be assumed to be proportional to the concentration of external photons. If we also assume weak absorption of external photons, the textured cell with a back mirror in column (3) is $4n^2$ times thinner than the planar cell with an absorbing substrate in column (3) so that both cases have the same absorption of external photons. We can then see that the voltage boost due to the addition of a texture and a back mirror corresponds to what we would expect from [12]; the voltage difference is $\Delta V = (kT/q)\ \ln\{4n^2\}$ between a planar solar cell with an absorbing substrate and a textured solar cell with a perfect back mirror.

III. Luminescence Extraction in Gallium Arsenide Cells

We have, thus far, looked at the limiting cases of η_{ext} for variously configured solar cells under the assumptions of optically very thick or thin material. In this section, we derive η_{ext} as a function of real material absorption, based on the physics of detailed balance, as in [1]. We then find the voltage penalty for high-quality planar GaAs cells and assess the benefits of a back mirror.

In the dark, under no illumination, blackbody radiation is absorbed by the cell through the front surface. We denote the blackbody radiation absorbed through the front as $b_\downarrow(E)$, with the downward arrow representing that the radiation is directed downwards through the front surface. We find $b_\downarrow(E)$ as follows:

$$\begin{aligned} b_\downarrow(E) &= \frac{2\pi E^2}{c^2 h^3 \left(\exp\left(\frac{E}{k_B T}\right) - 1\right)} A(E) \\ &\approx \frac{2\pi E^2}{c^2 h^3} \exp\left(\frac{-E}{k_B T}\right) A(E) \end{aligned} \tag{9}$$

where the blackbody spectrum $b_\downarrow(E)$ is in units of photons/area/time/energy, $A(E)$ is the absorptivity, E is the photon energy, c is the speed of light, h is Planck's constant, k_B is the Boltzmann constant, and T is the solar cell temperature.

The photons incident on the front surface show a Lambertian distribution. Upon entering the higher index semiconductor, they refract toward the normal. The absorptivity $A(E)$ is a Lambertian average over all incident angles:

$$\begin{aligned} A(E) &= 2\int_0^{\frac{\pi}{2}} \left(1 - \exp\left(-\frac{\alpha(E)\, L}{\cos\theta_2}\right)\right) \sin\theta \cos\theta\, d\theta \\ &\approx \left(1 - \exp\left(-\alpha(E)\, L\right)\right) \end{aligned} \tag{10}$$

where θ is the incident angle, $\theta_2 = \sin^{-1}\frac{\sin\theta}{n}$ is the angle inside the semiconductor, $\alpha(E)$ is the absorption coefficient as a function of photon energy, and L is the cell thickness. Since there is a large refractive index mismatch between air and the semiconductor, we can approximate $\cos\theta_2 \approx 1$.

In the dark, in thermal equilibrium, the emission from the front of the cell equals the absorption through the front. Under a potential *V*, the emission $L_{\text{ext}\uparrow}(E)$ takes the form

$$L_{\text{ext}\uparrow}(E) = \frac{2\pi E^2}{c^2h^3\left(\exp\left(\frac{E-qV}{k_BT}\right)-1\right)}A(E) \approx \frac{2\pi E^2}{c^2h^3}\exp\left(\frac{-E}{k_BT}\right)\exp\left(\frac{qV}{k_BT}\right)A(E) \quad (11)$$

where *q* is the charge of an electron.

The blackbody radiation absorbed from the back surface takes the form

$$b_\uparrow(E) = \frac{2\pi n^2E^2}{c^2h^3\left(\exp\left(\frac{E}{k_BT}\right)-1\right)}A_{\text{back}}(E) \approx \frac{2\pi n^2E^2}{c^2h^3}\exp\left(\frac{-E}{k_BT}\right)A_{\text{back}}(E) \quad (12)$$

where $A_{\text{back}}(E)$ is the absorptivity of the incident photons on the back surface, a Lambertian average over all incident angles. The factor of n^2 accounts for the increased density of states in the index-matched substrate. In the case where the substrate has been removed and there is an air interface, $b_\uparrow(E) = b_\downarrow(E)$. If an incident photon from the back is within the escape cone, it sees a single pass through the cell. If it is outside the escape cone, it sees a double pass, due to total internal reflection. Thus, we have

$$A_{\text{back}}(E) = 2\int_0^{\frac{\pi}{2}}\left(1-\exp\left(-\frac{f(\theta)\alpha(E)L}{\cos\theta}\right)\right) \times \sin\theta\cos\theta\, d\theta dE \quad (13)$$

where $f(\theta)$ is given as

$$f(\theta) = \begin{cases} 1, & \theta < \theta_c \\ 2, & \theta > \theta_c \end{cases} \quad (14)$$

where the critical angle θ_c is given by Snell's law as $\theta_c = \sin^{-1}\frac{1}{n}$.

Again, the absorption through the back of the cell is equivalent to the emission out the back of the cell in thermal equilibrium. Under a potential *V*, the emission out the back $L_{\text{ext}\downarrow}(E)$ is given as follows:

$$L_{\text{ext}\downarrow}(E) = \frac{2\pi n^2E^2}{c^2h^3\left(\exp\left(\frac{E-qV}{k_BT}\right)-1\right)}A_{\text{back}}(E) \approx \frac{2\pi n^2E^2}{c^2h^3}\exp\left(\frac{-E}{k_BT}\right)\exp\left(\frac{qV}{k_BT}\right)A_{\text{back}}(E). \quad (15)$$

Applying (4), we get the external luminescence yield η_{ext}, shown in (16) at the bottom of the page.

The total internal radiation $R_{\text{int}}(E)$, in units of photons/area/time/energy, can be given as

$$R_{\text{int}}(E) = \frac{8\pi n^2E^2}{c^2h^3\left(\exp\left(\frac{E-qV}{k_BT}\right)-1\right)}\alpha(E)L \approx \frac{8\pi n^2E^2}{c^2h^3}\exp\left(\frac{-E}{k_BT}\right)\exp\left(\frac{qV}{k_BT}\right)\alpha(E)L. \quad (17)$$

Photons that are not emitted out of the front surface or the back surface of the solar cell must be reabsorbed. We can, thus, write the reabsorbed radiation $R_{\text{abs}}(E)$, in units of photons/area/time/energy, as

$$R_{\text{abs}}(E) = R_{\text{int}}(E) - L_{\text{ext}\downarrow}(E) - L_{\text{ext}\uparrow}(E). \quad (18)$$

The probability that a luminescent photon is reabsorbed P_{abs} is thus given as

$$P_{\text{abs}} = \frac{\int_0^\infty R_{\text{abs}}(E)\,dE}{\int_0^\infty R_{\text{int}}(E)\,dE}. \quad (19)$$

After an incident photon is absorbed and reemitted radiatively, there are three options: 1) internal reabsorption, 2) emission out the back surface, or 3) emission out the front surface. Fig. 3 shows the fate of internal photons, as a function of photon energy. The total internal emission spectrum [see (17)] is normalized to a total area of unity, and the spectrum of reabsorbed photons [see (18)], emission out the back surface [see (15)], and emission out the front surface [see (11)] are also normalized by the same factor. Fig. 3(a) shows the spectrum of internal luminescence for a 1-μm-thick GaAs cell, and Fig. 3(b) concerns a 100-nm GaAs cell. In Fig. 3 and following figures, we assume the temperature of the cell $T = 30$ °C.

For the absorption coefficient of GaAs $\alpha(E)$, we use the fit by [1] to the data in [17], with one modification. The fit in [1] ignores the exciton bump at the bandedge. Here, we model this bump with a fourth-degree polynomial curve fit to the measured data in [17] (see Fig. 4).

In Fig. 3(a), we show the spectrum and breakdown of internal photons in a 1-μm-thick cell on index-matched substrate. The majority of internal photons are reabsorbed, a small amount is emitted into the index-matched substrate on the bottom, and an even smaller amount escapes out of the front surface. On the other hand, in Fig. 3(b), in the thinner 100-nm cell, the majority of photons are emitted out of the back surface and a small amount is reabsorbed (again, a tiny portion of photons escape out of the front surface of the cell). A thicker cell is more likely to reabsorb internal photons before they can escape. The

$$\eta_{\text{ext}} = \frac{\int_0^\infty L_{\text{ext}\uparrow}(E)\,dE}{\int_0^\infty L_{\text{ext}\uparrow}(E)\,dE + \int_0^\infty L_{\text{ext}\downarrow}(E)\,dE} = \frac{\int_0^\infty \frac{2\pi E^2}{c^2h^3}\exp\left(\frac{-E}{k_BT}\right)A(E)\,dE}{\int_0^\infty \frac{2\pi E^2}{c^2h^3}\exp\left(\frac{-E}{k_BT}\right)A(E)\,dE + \int_0^\infty \frac{2\pi n^2E^2}{c^2h^3}\exp\left(\frac{-E}{k_BT}\right)A_{\text{back}}(E)\,dE}. \quad (16)$$

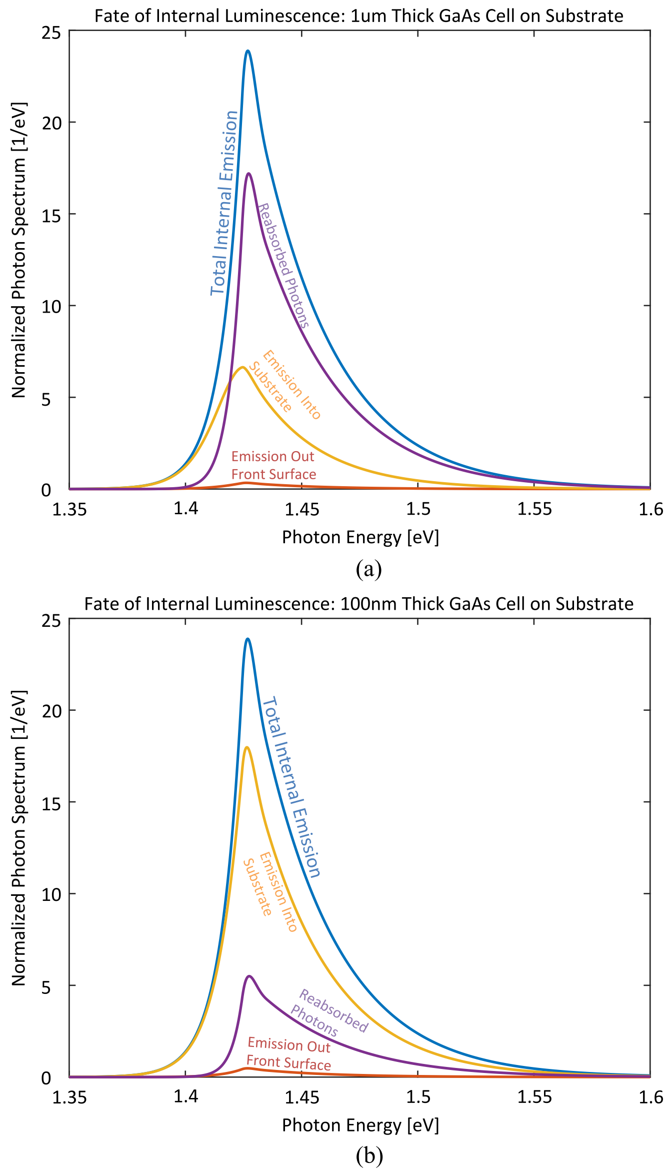

Fig. 3. Breakdown of internal photons in a GaAs cell, grown on a substrate for (a) a 1-μm-thick cell and (b) a 100-nm-thick cell. The spectrum is shown for total internal emission [see (17)], normalized to a total area of unity (blue). Of the total internal spectrum, the fraction of photons reabsorbed by the cell (purple), photons escaping out the front surface (red), and photons emitted in the substrate (yellow), are shown.

external luminescence yield η_{ext} is the ratio of the emission out the front surface to the total emission out of the front and back surfaces. From the spectra in Fig. 3(a) and (b), we can see that η_{ext} will be greater for the thicker 1um cell. The intuition for this is as follows: Each reabsorption event randomizes the angular direction of the internal photon. More reabsorption events allow the internal photon more chances to be in the escape cone and, thus, be reemitted out of the front surface. Thus, the thicker cell will have a higher η_{ext} in this ideal scenario of only radiative recombination.

For a high-efficiency solar cell, very high absorption of the above bandgap photons is required for high short-circuit current. Paradoxically, even though we may have almost step function absorption of incident photons, the cell is not necessarily

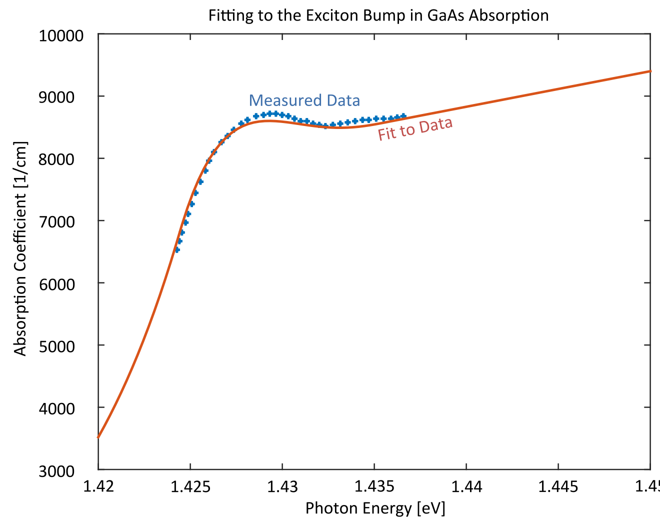

Fig. 4. Absorption coefficient as a function of photon energy, around the bandedge of GaAs. The blue crosses represent the data from [13], showing a bump around the bandedge due to absorption at the exciton energy. The absorption coefficient fit described in [1] ignores this exciton bump. We use the fit in [1], except at the bandedge, where we model the exciton bump with a fourth-degree polynomial. The red line shows our fit.

strongly absorbing of the internal luminescence. We can resolve this paradox by realizing that the internal luminescence is downshifted in energy from the incident photons. Due to the Urbach tail in the absorption spectra of many materials [18], a portion of this internal luminescence is even below the bandedge, where we see very weak absorption. We can see this in Fig. 3; the bandgap for GaAs is $E_g = 1.42$ eV [1], but we have internal photons with an energy lower than the bandgap. Consequently, such as in [15], it is reasonable to assume both step function absorption and weak reabsorption of internal luminescence.

In Fig. 5, we plot the voltage penalty $\Delta V_{\text{penalty}}$ for a GaAs cell on an index-matched substrate, as a function of the reabsorption probability P_{abs} (each value of P_{abs} corresponds to a certain GaAs cell thickness L which is also denoted on the graph). We see from Fig. 5 that for a 1-μm cell on substrate, $\Delta V_{\text{penalty}} = -81$ mV, and for a 100-nm cell on substrate, $\Delta V_{\text{penalty}} = -96$ mV. In the limit of a cell that is optically thin to the internal luminescence ($P_{\text{abs}} \to 0$), we see that $\Delta V_{\text{penalty}} = -101$ mV in Fig. 5, which is consistent with (1) and (3). In the limit of a cell that is optically thick to the internal luminescence ($P_{\text{abs}} \to 1$), we see that $\Delta V_{\text{penalty}} = -68$ mV, which is consistent with (1) and (6). To reach the limit of a cell that is optically thick to the internal luminescence ($P_{\text{abs}} \to 1$), we must have a cell that is infinitely thick.

In Fig. 5, we plot horizontal reference lines for $\Delta V_{\text{penalty}} = 0$, the case of a perfect back reflector, and $\Delta V_{\text{penalty}} = -13.6$ mV, the case of an air interface ($n = 1$) at the back of the cell. To find $\Delta V_{\text{penalty}}$ for the case of a semiconductor/air interface at the back surface, we use (16), modifying $A_{\text{back}}(E)$ to

$$A_{\text{back, air}}(E) = 2\left(1 - \exp\left(-\alpha(E)\,L\right)\right) \times \int_0^{\frac{\pi}{2}} \left(1 - R(\theta)\right) \sin\theta \cos\theta \, d\theta \qquad (20)$$

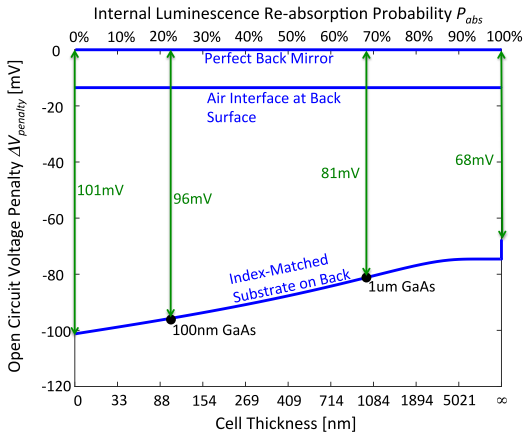

Fig. 5. Voltage penalty $\Delta V_{\mathrm{penalty}}$ for a GaAs cell on an index-matched substrate, as a function of the reabsorption probability P_{abs}, as given by (1), (16), and (19). Each value of P_{abs} corresponds to a certain cell thickness L, which is denoted on the bottom x-axis of the graph. Horizontal lines indicate where $\Delta V_{\mathrm{penalty}} = 0$, for the case of a perfect back reflector, and where $\Delta V_{\mathrm{penalty}} = -13.6$ mV, for the case of an air interface ($n = 1$) at the back of the cell. For a 1-μm cell on substrate, $\Delta V_{\mathrm{penalty}} = -81$ mV, and for a 100-nm cell on substrate, $\Delta V_{\mathrm{penalty}} = -96$ mV. In the limit of a cell that is optically thin to the internal luminescence ($P_{\mathrm{abs}} \rightarrow 0$), $\Delta V_{\mathrm{penalty}} = -101$ mV, consistent with our derived lower bound for η_{ext}. In the limit of a cell that is optically thick to the internal luminescence ($P_{\mathrm{abs}} \rightarrow 1$), we see that $\Delta V_{\mathrm{penalty}} = -68$ mV, consistent with our derived upper bound for η_{ext}.

where $R(\theta)$ is the reflectivity at the air/semiconductor interface. The Lambertian angle averaged transmissivity $\int_0^{\frac{\pi}{2}} (1 - R(\theta)) \sin\theta \cos\theta \, d\theta = 68\%$. Thus, $\Delta V_{\mathrm{penalty}} = -13.6$ mV by (1) and (16). It should be noted that though the front surface is also an interface to air, we assume an antireflection coating on the front; therefore, $A_{\mathrm{back,\ air}}(E) \neq A(E)$.

In Fig. 5, $\Delta V_{\mathrm{penalty}}$ is given in reference to $V_{\mathrm{oc,\ ideal}}$ for the three cases of a planar cell on an absorbing substrate, air back interface, and perfect back mirror. As shown in (7), $V_{\mathrm{oc,\ ideal}}$ is a function of $A(E)$; therefore, when comparing $\Delta V_{\mathrm{penalty}}$, care must be taken to compare cells with the same $A(E)$. For example, Fig. 5 plots $\Delta V_{\mathrm{penalty}}$ as a function of thickness of a GaAs cell on substrate. The 1-μm-thick GaAs cell on substrate should be compared with a 500-nm cell on a perfect back mirror, and a 100-nm-thick GaAs cell on substrate should be compared with a 50-nm cell on a perfect back mirror. In order to compare with a textured cell, (8) should be used to find comparable thicknesses, although for thin cells, this expression breaks down [19].

IV. Conclusion

At open-circuit voltage, we want to maximize the photon emission out of the front surface of a solar cell. We can maximize the external luminescence yield η_{ext} by putting a mirror on the backside of the cell and by surface texturing. However, there has been ambiguity over what factor of 2 increase η_{ext} gains from better luminescence extraction. In this study, we resolve factors of 2, finding that the increases in η_{ext}, and, consequently, voltage depend on whether the cell is strongly absorbing (optically thick) or weakly absorbing (optically thin) toward the internal luminescence.

In particular, we explored the case of untextured thin-film GaAs solar cells. For this particular case, η_{ext} is found to be bounded between $\approx \frac{1}{4n^2}$ and $\approx \frac{1}{n^2}$ for a cell on an index-matched substrate, for the limits of being optically thin and optically thick to the internal luminescence, respectively. For GaAs, with index of refraction $n = 3.5$, these expressions for η_{ext} correspond to open-circuit voltage penalties of $\Delta V_{\mathrm{penalty}} = -101$ mV and $\Delta V_{\mathrm{penalty}} = -68$ mV, respectively. Resolving this factor of 4 discrepancy in η_{ext} allows us to see what voltage benefit we will have when replacing an index-matched back substrate with a back mirror. We find that it is unrealistic to use the strong absorption limit of $\eta_{\mathrm{ext}} \approx \frac{1}{n^2}$, as coming close to this limit requires a GaAs cell of infinite thickness. This is due to the Urbach tail at the bandedge, which means that a portion of the internal luminescence will have energy below the bandedge, where GaAs is very weakly absorbing. For a 100-nm GaAs cell, we are close to the optically thin limit for the internal luminescence, with $\Delta V_{\mathrm{penalty}} = -96$ mV. This means that when we replace a back substrate with a back mirror and halve the thickness of the cell to keep the absorption constant, it is possible to pick up 96 mV in open-circuit voltage. For thicker cells, the voltage penalty decreases, with a 1-μm-thick cell having $\Delta V_{\mathrm{penalty}} = -81$ mV.

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Authors’ photographs and biographies not available at the time of publication.